16.8. Divergence Theorem

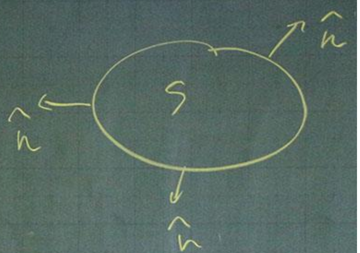
$$\int_V \nabla\cdot\vec{V}\,dV = \int_S \hat{n}\cdot\vec{V}\,dA$$

$$\int_V \nabla\times\vec{V}\,dV = \int_S \hat{n}\times\vec{V}\,dA$$

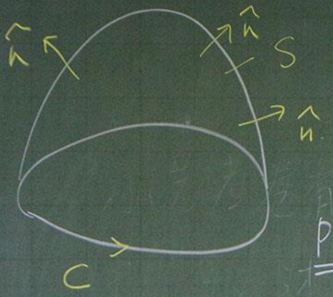
$$\int_V \nabla u\,dV = \int_S u\,dA$$

2D Divergence Theorem

$$\int_S \nabla\cdot\vec{v}\,dA = \int_C \underline{\underline{\hat{n}}}\cdot\vec{v}\,dS$$

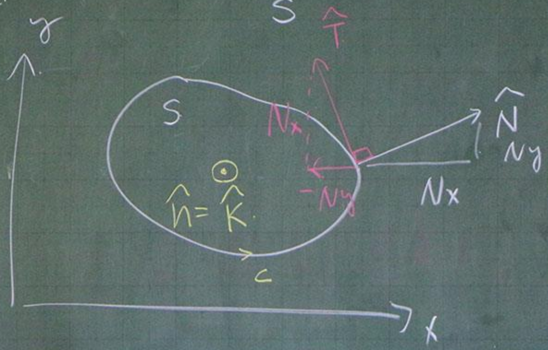


16.7. Stokes's Theorem.



pt

$$\oint_C \vec V\cdot d\vec R = \int_S \hat n\cdot(\nabla\times\vec V)\,dA$$



$\hat N = N_x\hat i + N_y\hat j$; $\hat T = -N_y\hat i + N_x\hat j$ ———— 1(a,b)

$\vec V = V_x\hat i + V_y\hat j$; set $\vec u = u_x\hat i + u_y\hat j$

$= -V_y\hat i + V_x\hat i$ (i.e., $u_x = -V_y$, $u_y = V_x$)

———— 2(a,b)

16.8 Div
Theorem

16.9 Stok
Theorem

16.10 Irro
-onal fi

Midterm
(Thur

Let's apply 2D divergence theorem.

$$\int_S \nabla\cdot\vec{V}\, dA = \int_C \hat{n}\cdot\vec{V}\, dS \qquad (3)$$

LHS: $\displaystyle \int_S \underline{\nabla\cdot\vec{V}}\, dA = \int_S \left(\frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y}\right) dA$

$$= \int_S \left(\frac{\partial U_y}{\partial x} - \frac{\partial U_x}{\partial y}\right) dA$$

critical: $\displaystyle \int_S \hat{k}\cdot \underline{\nabla\times\vec{U}}\, dA \qquad (4)$

PS

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ U_x & U_y & 0 \end{vmatrix} \rightarrow \hat{k}\left(\frac{\partial U_y}{\partial x} - \frac{\partial U_x}{\partial y}\right)$$

$$\text{RHS} = \int_C \hat{n}\cdot\vec{V}\,dS = \int_C (N_x\hat{i}+N_y\hat{j})\cdot(V_x\hat{i}+V_y\hat{j})\,dS = \int_C (N_x \underline{V_x} + N_y \underline{V_y})\,dS$$

$$= \int_C (N_x \underline{U_y} - N_y \underline{U_x})\,dS = \int_S (U_x\hat{i}+U_y\hat{j})\cdot(-N_y\hat{i}+N_x\hat{j})\,dS = \int_S \vec{U}\cdot\hat{T}\,dS$$

$$= \int_S \vec{U}\cdot d\vec{R} \quad \text{——— (5)}$$

$$\text{LHS} = \text{RHS}\ ; \quad (4) = (5)$$

$$\int_S \hat{k}\cdot\nabla\times\vec{u}\,dA = \int \vec{u}\cdot d\vec{R}$$

$$\Downarrow$$

$$\int_S \hat{n}\cdot\nabla\times\vec{u}\,dA = \int \vec{u}\cdot d\vec{R} \quad \#$$

16.8 Divergence Theorem

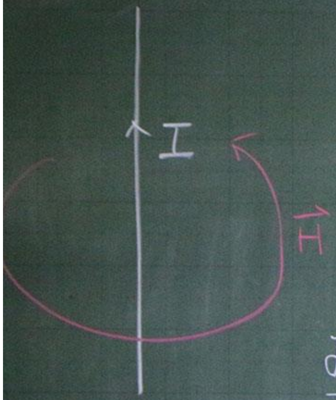
16.9 Stokes' Theorem

16.10 Irrotational field

Midterm #1 (Thur.)

<Ex.>

$$\oint \vec{H}\cdot d\vec{R} = I \quad \text{Ampere's Law} \quad (*)$$



$$\to \nabla\times\vec{H} = \vec{J}$$

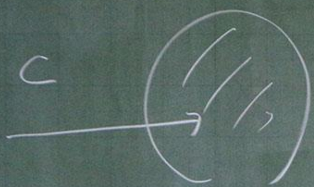
Pf

$$\text{LHS} = \oint_C \vec{H}\cdot d\vec{R}$$

$$= \int_S \hat{n}\cdot(\nabla\times\vec{H})\,dA \quad \text{—— (1)}$$

$$\text{RHS} = \underset{\downarrow \atop \frac{\text{Coul}}{s}}{I} = \int \hat{n}\cdot\underset{\downarrow \atop \frac{C}{m^2\cdot s}}{\vec{J}}\,dA = \int \hat{n}\cdot(\sigma\vec{v})\,dA$$

$$\frac{C}{m^3}\cdot\frac{m}{s} = \frac{C}{m^2\cdot s} \quad \text{—— (2)}$$



(1) = (2).

$$\int_S \hat{n}\cdot(\nabla\times\vec{H})\,dA = \int \hat{n}\cdot\vec{J}\,dA$$

$$\to \int_S \hat{n}\cdot[\nabla\times\vec{H}-\vec{J}]\,dA = 0$$

$\therefore \ \nabla \times \vec{H} = \vec{J}$

integral form $\xrightarrow{\text{Ch. 16}}$ differential form

4 laws

1. Divergence
2. Stoke's

$\downarrow$

Wave Eqn.

$$\nabla \times \nabla \times \vec{E} = \nabla(\nabla \cdot \vec{E}) - \nabla^2 \vec{E}$$

16.10 Irrotational fields (非旋場)

I. $\nabla \times \vec{V} = 0$

II. a. There exists a C^2 scalar fxn. ϕ in D, such that $\nabla \phi = \vec{V}$

b. $\nabla \times \vec{V} = 0$

c. $\oint_C \vec{V} \cdot d\vec{R} = 0$, d. $\int_C \vec{V} \cdot d\vec{R}$ is independent w/ C

16.8 Divergence Theorem

16.9 Stokes's Theorem

16.10 Irrotational field

Midterm #1 (Thur.)

(a) → (b)

$$\nabla \phi = \frac{\partial \phi}{\partial x}\hat{i} + \frac{\partial \phi}{\partial y}\hat{j} + \frac{\partial \phi}{\partial z}\hat{k}$$

$$= \phi_x \hat{i} + \phi_y \hat{j} + \phi_z \hat{k}$$

$$= V_x \hat{i} + V_y \hat{j} + V_z \hat{k}$$

$$\nabla \times \vec{V} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \phi_x & \phi_y & \phi_z \end{vmatrix}$$

$$= \hat{i}\,\underbrace{(\phi_{zy} - \phi_{yz})}_{=0} + \hat{j}\,\underbrace{(\phi_{xz} - \phi_{zx})}_{=0} + \hat{k}\,\underbrace{(\phi_{yx} - \phi_{xy})}_{=0} = 0$$

(b) → (c)

$$\oint_C \vec{V} \cdot d\vec{R} \xlongequal[\text{theorem}]{\text{stoke's}} \int \hat{n} \cdot (\underbrace{\nabla \times \vec{V}}_{0})\, dA$$

$$= 0$$

$$= \hat{i}\,\underbrace{(\phi_{zy} - \phi_{yz})}_{0} + \hat{j}\,\underbrace{(\phi_{xz} - \phi_{zx})}_{0} + \hat{k}\,\underbrace{(\phi_{yx} - \phi_{xy})}_{0} = 0$$

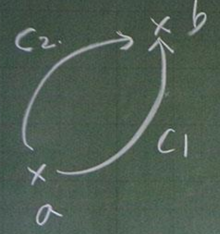
(b) $\rightarrow$ (c)

$$\oint_C \vec{V}\cdot d\vec{R} \overset{\text{stoke's}}{\underset{\text{theorem}}{=}} \int \hat{n}\cdot(\underbrace{\nabla \times \vec{V}}_{0})\, dA$$

$$= 0$$

(c) $\rightarrow$ (d)

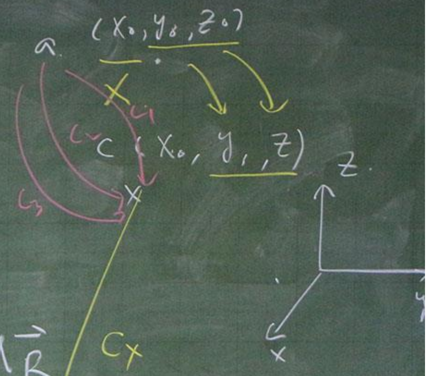
$C_1 - C_2 = C$



$$\oint_C \vec{V}\cdot d\vec{R} = 0 = \int_{C_1} \vec{V}\cdot d\vec{R} - \int_{C_2} \vec{V}\cdot d\vec{R} \quad \therefore \int_{C_1} \vec{V}\cdot d\vec{R} = \int_{C_2} \vec{V}\cdot d\vec{R}$$

#.

(d) → (a)



$$\int_a^b \vec{V}\cdot d\vec{R} = \phi(x, y, z)$$

Single-valued ——— (1)

$$\int \vec{V}\cdot d\vec{R} = \int_{C_1} \vec{V}\cdot d\vec{R} + \int_{C_X} \vec{V}\cdot d\vec{R}$$

$$= \int_{C_2} \vec{V}\cdot d\vec{R} + \int_{C_X} \vec{V}\cdot d\vec{R} = \cdots$$

b $(\underbrace{x}_{V}, \underbrace{y, z}_{X})$ ——— (2)

$$\int_{C_1} \vec{V}\cdot d\vec{R} = \int_{C_2} \vec{V}\cdot d\vec{R} = \int_{C_2} \vec{V}\cdot d\vec{R} = \text{const.}$$

——— (3)

(1), (2) lead to,

$$\int \vec{V}\cdot d\vec{R} = \phi = \text{const.} + \int_{C_X} \vec{V}\cdot d\vec{R}$$

$$= C + \int_{C_X} V_x \, dx$$

$$d\vec{R} = dx\,\hat{i} + dy\,\hat{j} + dz\,\hat{k} = dx\,\hat{i}$$

$$= c + \int_{x_0}^{x} V_x(\xi, y, z)\, d\xi$$

$$\frac{\partial \phi}{\partial x} = V_x(x, y, z)$$

By repeating this process, we obtain,

$$\frac{\partial \phi}{\partial y} = V_y, \quad \frac{\partial \phi}{\partial z} = V_z \Rightarrow \nabla \phi = \vec{V}$$

#

$$\int_C \vec{V} \cdot d\vec{R} = \phi \Big|_a^b$$

$$\nabla \phi = \vec{V}$$

<conclusion>

$$\int_C \vec{V} \cdot d\vec{R} \xrightarrow{\nabla \times \vec{V} = 0}$$

1. $\nabla \times \vec{V} = 0$

$\oint_C = 0$ $\qquad$ $\int_C \sim = ?$

$$\int_C \vec{V}\cdot d\vec{R} = \phi(x,y,z)\Big|_{P_i}^{P_f}$$

$$\nabla\phi = \vec{V}$$

2. $\nabla\times\vec{V} \neq 0$; $\int_C \vec{V}\cdot d\vec{R}$

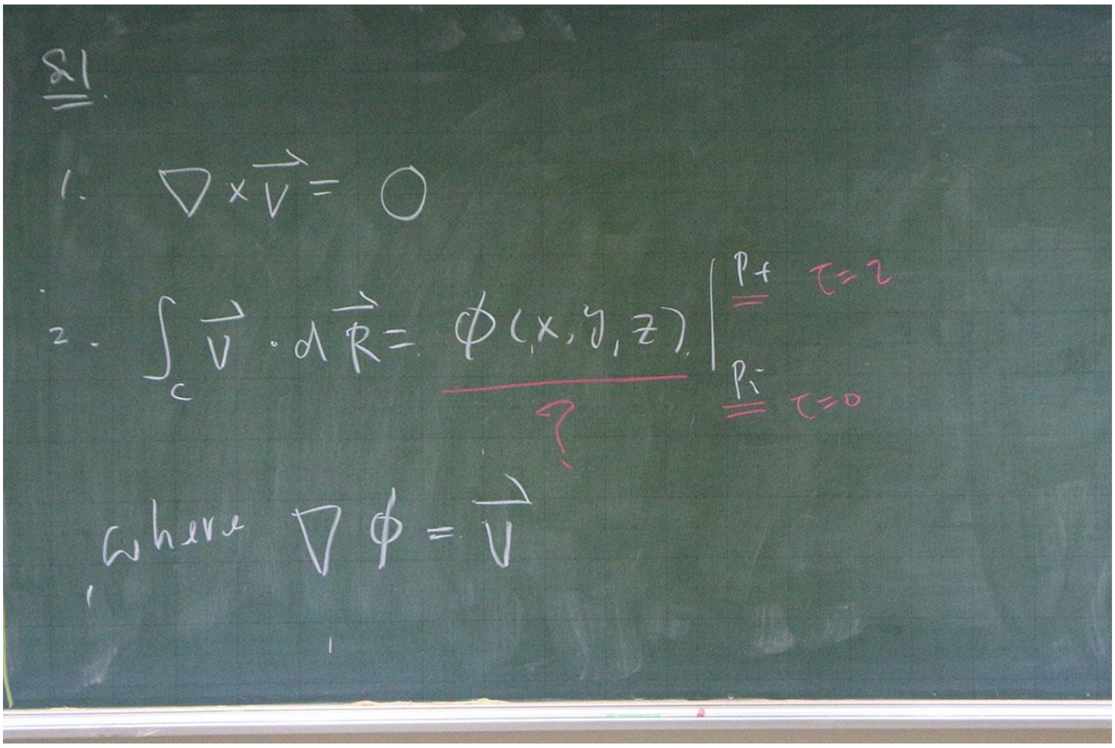
<Ex.>

$I = \int_C \vec{V}\cdot d\vec{R}$, where $\vec{V} = 3x^2y^2\,\hat{i} + (2x^3y - e^z)$

$+ (2z - ye^z)\,\hat{k}$, and C is, $x(\tau) = (1-\frac{\tau}{2})$

$\cdot\, e^{\sqrt{\tau}}$, $y(\tau) = \frac{-6}{\tau+3} + \frac{21}{10}\tau$,

$z(\tau) = \frac{1}{16}\tau^5 - 1$, for $\tau : 0 \to 2$


§1
1. $\nabla \times \vec{V} = 0$
2. $\int_c \vec{V} \cdot d\vec{R} = \phi(x,y,z) \Big|_{P_i}^{P_f}$
?
$t=2$
$t=0$
where $\nabla \phi = \vec{V}$


Namely
$\frac{\partial \phi}{\partial x} = V_x = 3x^2y^2 \rightarrow \phi(x,y,z) = \int V_x \, dx$
$+ A(y,z) = x^3y^2 + A(y,z)$ —— (2)
$\frac{\partial \phi}{\partial y} = V_y = 2x^3y - e^z \rightarrow \phi =$
$+ B$

Namely

$$\frac{\partial \phi}{\partial x} = V_x = 3x^2y^2 \longrightarrow \phi(x,y,z) = \int V_x \, dx$$

$$+ A(y,z) = x^3y^2 + A(y,z) \qquad (2)$$

$$\frac{\partial \phi}{\partial y} = V_y = 2x^3y - e^z \Longrightarrow \phi = \int V_y \, dy + B$$